



# Projectile Dynamics under Aerodynamic Drag: Application to the Flight of a Soccer Ball

Research Article

<https://stem.techspherejournal.com>

## Article Info

Revised Date: 28<sup>th</sup> August, 2025

Accepted Date: 5<sup>th</sup> September, 2025

Published Date: 11<sup>th</sup> September, 2025

## Author Details

Haruna Mamman<sup>1\*</sup>, Ekene Kevin Ikeme<sup>2</sup>, Ojo Opeyemi Olukayode<sup>3</sup>  
1, 2, 3 Department of Science and Lab Technology, Federal Polytechnic Ado-Ekiti,  
Ekiti State.

## Keywords

Projectile

Aerodynamic

Soccer Ball

Magnus Force

Trajectory

\*Corresponding author's email: [haruna\\_ma@fedpolyado.edu.ng](mailto:haruna_ma@fedpolyado.edu.ng)

DOI: <https://doi.org/10.5281/zenodo.17102567>

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## ABSTRACT

Projectile motion is traditionally introduced in physics through the parabolic trajectories of idealised objects moving under uniform gravity. However, real-world projectiles such as soccer balls deviate significantly from this model due to the influence of aerodynamic forces. This study investigates the dynamics of a soccer ball in free-kick scenarios by progressively incorporating quadratic drag and the Magnus effect into the equations of motion. A theoretical framework was developed using Newton's second law, where resistive drag was modelled as proportional to the square of velocity and spin-induced lift was incorporated through the Magnus effect. Numerical integration methods were employed to solve the coupled nonlinear differential equations governing the ball's trajectory. Simulations were conducted for varying launch angles, initial velocities and spin rates, enabling comparisons between idealized and aerodynamically modified flight paths. Results reveal that aerodynamic drag significantly reduces range and alters the optimal launch angles from the theoretical  $45^\circ$  to approximately  $20^\circ - 25^\circ$ , in agreement with observations from professional soccer. The Magnus effect introduces lateral deviations that reproduce the curved trajectories seen in bending free kicks, with the magnitude of deflection dependent on both spin rate and initial velocity. The analysis underscores the importance of aerodynamic modelling in bridging the gap between idealized mechanics and practical sports applications.

## **1 Introduction**

Projectile motion has long served as a foundational topic in classical mechanics, offering an elegant demonstration of Newton's laws of motion and the principles of kinematics. In its idealized form, projectile motion assumes the absence of resistive forces, allowing for analytical solutions that describe parabolic trajectories under uniform gravity. While such a model is sufficient for introductory physics, it often fails to capture the dynamics of real-world projectiles where additional forces act on the body during flight (Wadsworth et al., 2024; Walley, 2018).

One of the most significant departures from the idealized case arises when aerodynamic forces are included. In particular, air resistance (often modelled as quadratic function of velocity) plays a crucial role in altering both the range and height of the projectile. This effect is especially relevant in sports physics, where objects such as balls move at relatively high speeds and over distances where drag cannot be neglected (Owen & Ryu, 2005).

Beyond drag, many sports projectiles exhibit lateral deviations due to the Magnus effect, a force arising from the interaction of spin with the surrounding airflow. This aerodynamic lift force is responsible for the curved flight paths



observed in soccer free kicks, baseball pitches and tennis shots (Kray et al., 2014). In soccer for example, skilled players exploit spin to bend the ball around defensive walls and toward the goal, a phenomenon that cannot be explained within the framework of ideal projectile motion.

The study of soccer ball trajectories thus provides both a compelling example of physics in everyday life and a practical application of advanced mechanics. By incorporating drag and lift forces into the analysis, a more realistic description of projectile dynamics can be achieved. Such models are not only of pedagogical value in bridging classroom theory with real-world phenomena, but they also offer insights into the optimization of techniques in sports (Zi & Gao, 2023). The present work aims to develop and analyze a simplified model of soccer ball flight under the influence of aerodynamic drag and spin-induced lift. Numerical methods are employed to solve the equations of motion and trajectories are compared for different initial conditions. The study highlights the differences between ideal and non-ideal projectile motion and demonstrates how aerodynamic forces govern flight of a spinning soccer ball.

### 1.1 Theoretical Framework

The motion of a projectile in the presence of aerodynamic forces is governed by Newton’s second law of motion, expressed as

$$m \frac{d\vec{v}}{dt} = \sum \vec{F} \dots\dots\dots 1$$

Where  $m$  is the mass of the projectile,  $\vec{v}$  is the velocity and  $\sum \vec{F}$  represents the sum of forces acting on the body (Alexander, 2017). For the case of a soccer ball in flight, the relevant forces include gravitational force, aerodynamic drag and the Magnus (lift) force due to spin.

#### 1.1.1 Ideal Projectile Motion

In the absence of resistive forces, the only force acting on the ball (apart from the normal reaction at launch) is gravity. The equations of motion in the horizontal (x) and vertical (y) directions are:

$$x(t) = v_0 \cos \theta t \dots\dots\dots 2$$

$$y(t) = v_0 \sin \theta t - \frac{1}{2} g t^2 \dots\dots\dots 3$$

Where  $v_0$  is the initial launch velocity,  $\theta$  is the launch angle and  $g$  is the acceleration due to gravity (Packel & Yuen, 2004). This results in a parabolic trajectory with a maximum range given by:

$$R = \frac{v_0^2 \sin 2\theta}{g} \dots\dots\dots 4$$

#### 1.1.2 Aerodynamic Drag Force

In reality, a soccer ball moving through air experiences resistive forces due to the viscosity and density of the surrounding fluid. At the typical speeds of a soccer free kick (20-30 m/s), the Reynolds number is sufficiently high that drag can be modelled as proportional to the square of the velocity (Robinson & Robinson, 2013). The drag is given by:

$$F_d = \frac{1}{2} \rho C_d A v^2 \dots\dots\dots 5$$

Where:  $\rho$  is the air density

$C_d$  is the drag coefficient (dimensionless),

$A = \pi r^2$  is the cross-sectional area of the ball,

$v$  is the instantaneous velocity magnitude

The drag force always acts opposite to the direction of motion. The equations of motion become coupled nonlinear differential equations:

$$m \frac{dv_x}{dt} = -\frac{1}{2} \rho C_d A v v_x \dots\dots\dots 6$$



$$m \frac{dv_y}{dt} = -mg - \frac{1}{2} \rho C_d A v v_y \dots\dots\dots 7$$

Where  $v_x$  and  $v_y$  are the velocity components (Lubarda & Lubarda, 2022).

Magnus (Lift) Force Due to Spin

When the soccer ball is imparted with spin, the boundary layer of airflow is deflected, producing a lateral force perpendicular to the velocity vector. The Magnus force describes this:

$$F_l = \frac{1}{2} \rho C_l A v^2 \dots\dots\dots 8$$

Where  $C_l$  is the lift coefficient, dependent on the spin rate and ball surface properties.

If the spin is about the vertical axis, the lift force acts in the horizontal z-direction, leading to curved trajectories (“the banana kick” in soccer). The modified equations of motion are then:

$$m \frac{dv_x}{dt} = -\frac{1}{2} \rho C_d A v v_x \dots\dots\dots 9$$

$$m \frac{dv_y}{dt} = -mg - \frac{1}{2} \rho C_d A v v_y \dots\dots\dots 10$$

$$m \frac{dv_z}{dt} = \frac{1}{2} \rho C_l A v^2 \dots\dots\dots 11$$

Where x denotes the lateral axis perpendicular to the plane of the initial kick.

Typical Parameter Values

For a regulation soccer ball:

1. Mass  $m \approx 0.43 \text{ kg}$ ,
2. Radius  $r \approx 0.11\text{m}$ ,
3. Drag coefficient  $C_d \approx 0.25\text{-}0.35$  depending on flow conditions,
4. Lift coefficient  $C_l \approx 0.1\text{-}0.3$  depending on spin rate,
5. Air density  $\rho \approx 1.2 \text{ kg/m}^3$  at sea level (Hall, 2018)

## 2 Methodology

The mathematical framework outlined in Section 2 provides the basis for modelling the flight of a soccer ball under aerodynamic forces. Since the governing equations are nonlinear and coupled, analytical solutions are generally not possible. Numerical methods are therefore required to simulate trajectories under different initial conditions.

### 2.1 Governing Equations

The equations of motion, including drag and Magnus forces, are:

$$m \frac{dv_x}{dt} = -\frac{1}{2} \rho C_d A v v_x \dots\dots\dots 12$$

$$m \frac{dv_y}{dt} = -mg - \frac{1}{2} \rho C_d A v v_y \dots\dots\dots 13$$

$$m \frac{dv_z}{dt} = \frac{1}{2} \rho C_l A v^2 \dots\dots\dots 14$$

With velocity magnitude defined as:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \dots\dots\dots 15$$

These equations describe motion in three dimensions, with the x-axis along the direction of the kick, the y-axis vertical and the z-axis lateral to the initial plane of the kick (Kajiyama & Yuji., 2024).



## 2.2 Numerical Method

Due to the nonlinear terms ( $vv_x, vv_y$ ), closed-form solutions are not available except in simplified cases. The equations were therefore solved using a numerical integration scheme. A fourth-order Runge-Kutta (RK4) method was employed because of its balance between computational efficiency and accuracy (Chauhan & Srivastava, 2019).

The system of differential equations was discretized over a small-time step  $\Delta t$  (typically 0.001-0.01 s), and the position and velocity components were updated iteratively. The integration proceeded until the vertical coordinate ( $y$ ) returned to zero, corresponding to the ball contacting the ground.

## 2.3 Initial Conditions

Simulations were conducted with parameters representative of professional soccer kicks:

1. Initial velocity:  $v_0 = 25\text{m/s}$  ( $\approx 90\text{ km/h}$ )
2. Launch angle:  $\theta = 20^\circ$  above horizontal
3. Spin rate:  $\omega = 6 - 10\text{ rps}$  (depending on the scenario)
4. Ball mass:  $m = 0.43\text{kg}$
5. Radius:  $r = 0.11\text{m}$
6. Drag coefficient:  $C_d = 0.25 - 0.35$
7. Lift coefficient:  $C_l = 0.1 - 0.3$
8. Air density:  $\rho = 1.2\text{ kg/m}^3$

The spin rate was applied about the vertical axis to induce lateral curvature.

## 2.4 Simulation Scenarios

Three cases were examined to isolate the effects of drag and lift:

1. Ideal projectile motion - no aerodynamic forces, gravity only.
2. With drag only – quadratic air resistance included, no spin.
3. With drag and Magnus force - both aerodynamic drag and lift included.

Comparisons across these cases provide insight into the influence of resistive and lift forces on soccer ball trajectories.

## Computational Tools

The equations were solved using Python with standard numerical libraries (Numpy, Scipy). Visualization was performed with Matplotlib to produce 2D and 3D trajectory plots.

## 3 Results and Discussion

### 3.1 Ideal Projectile Motion

In the absence of aerodynamic forces, the soccer ball follows a parabolic trajectory as predicted by classical mechanics. For an initial velocity  $v_0=25\text{ m/s}$ , a launch angle of  $\theta=20^\circ$ , the maximum height reached is approximately  $H \approx 5.3\text{m}$  and the horizontal range is  $R \approx 60\text{m}$ . Such results are consistent with textbook equations of projectile motion.

However, in practice, professional players rarely achieve such distances when kicking a soccer ball, highlighting the need to include aerodynamic effects.

### 3.2 Trajectory with Aerodynamic Drag

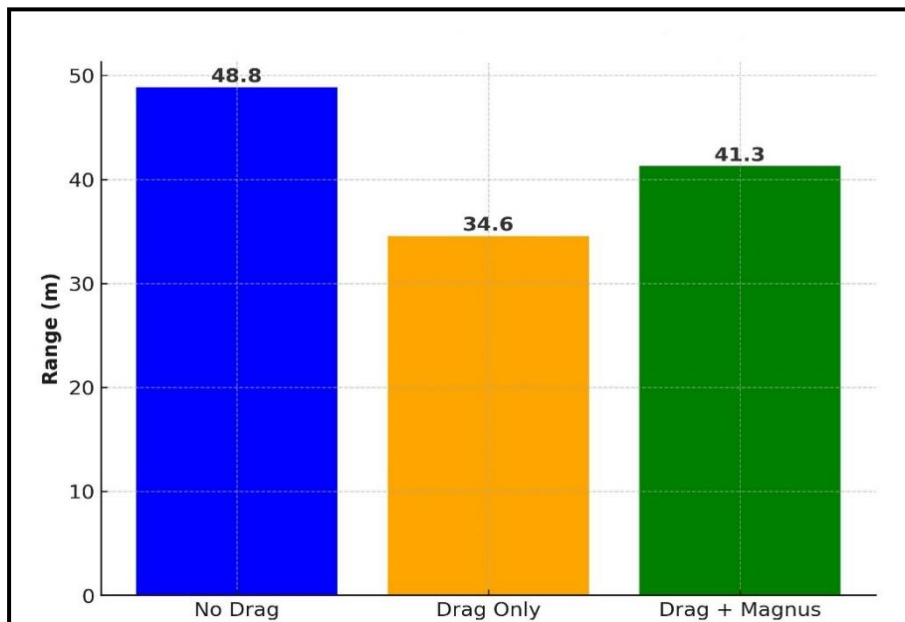
When quadratic drag is included, the ball's horizontal range is significantly reduced. Numerical simulations indicate that under typical conditions  $C_d=0.3, \rho=1.2\text{ kg/m}^3$ , the range decreases from  $R \approx 60\text{m}$  (ideal case) to about  $R \approx 35\text{m}$ .

Similarly, the maximum height is reduced because drag dissipates vertical velocity as well. The simulation results are presented in Table 1.

**Table 1:** Range in metres comparison between no drag, drag only and drag + Magnus

Trajectory Case	Range (m)
No Drag	48.8
Drag Only	34.6
Drag + Magnus	41.3

This result emphasizes that drag is the dominant resistive force in projectile dynamics at moderate to high speeds (Parker, 1977). In soccer, this explains why long-distance kicks quickly lose altitude despite high launch velocities.

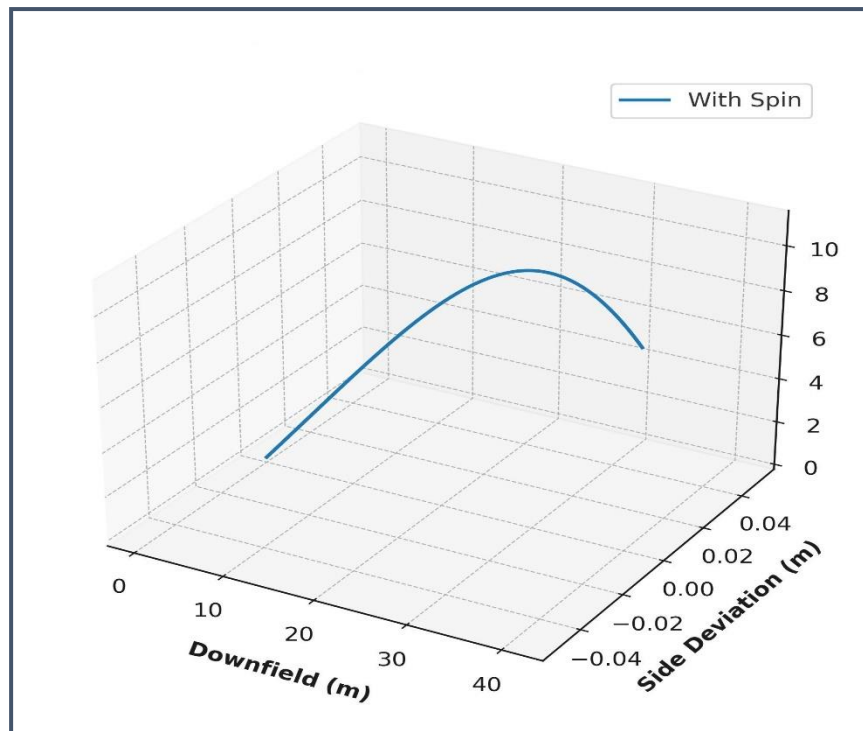


**Figure 1:** Comparison of trajectories with and without drag in the x-y plane

### 3.3 Trajectory with Drag and Magnus Force

When spin is applied to the ball, the Magnus effect introduces a lateral force, causing the ball to curve out of the original vertical plane of motion. Simulations show that for spin rates of  $\omega=8$  rps, lateral deviations of up to 3-5 meters can be obtained over a 30-meter free kick.

This magnitude of deviation is sufficient to bend the ball around a defensive wall and back toward the goal, corresponding well with observations of professional free kicks. Increasing the spin rate enhances curvature, though excessive spin can reduce forward distance due to increased drag-lift interaction.

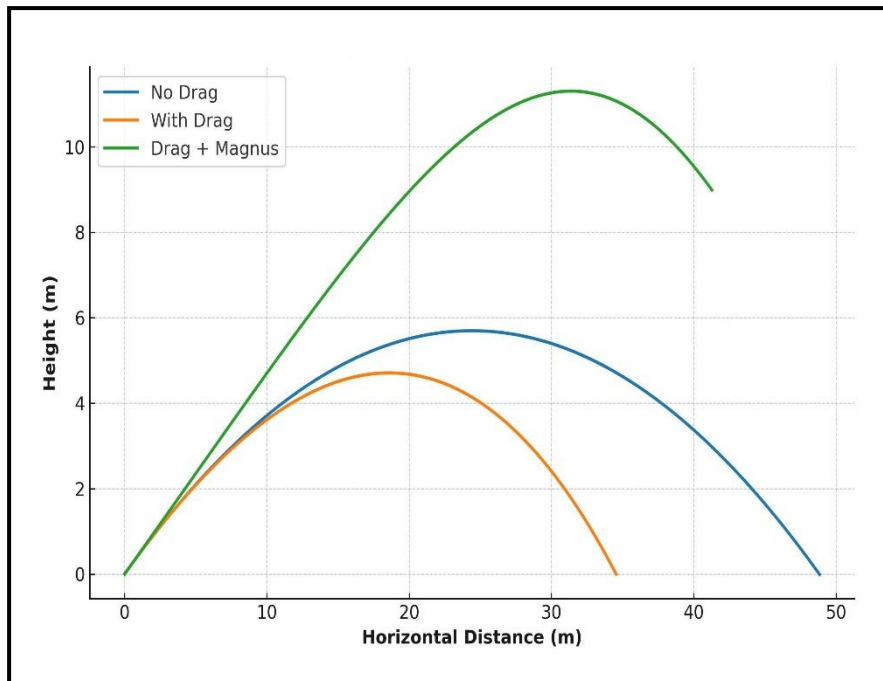


**Figure 2:** 3D trajectory of a spinning ball, showing lateral deviation in the x-z plane

### 3.4 Effect of Launch Angle and Spin

The relationship between launch angle and range changes significantly once drag is included. While the ideal case predicts a maximum range at  $\theta=45^\circ$ , the presence of drag shifts the optimal angle downward, typically to around 20-25 (Aziz & Bylbyl, 2019). This aligns with the angles most often observed in soccer free kicks.

Spin also modifies trajectory optimization. At low spin rates, drag dominates and limits range. At moderate spin rates, Magnus lift can extend flight time slightly while curving the ball laterally. At very high rates, energy losses due to increased drag offset any advantage.



**Figure 3:** Range vs launch angle for ideal, drag-only and drag + Magnus cases

### 3.5 Physical and Pedagogical Implications

From a physical standpoint, the analysis confirms that aerodynamics forces cannot be neglected in modelling sports projectiles. Drag is primarily responsible for reducing the effective range, while Magnus effect explains the lateral deviation exploited in “bending” free kicks.

From an educational perspective, the soccer ball provides an engaging real-world example to demonstrate the transition from idealized textbook physics to applied mechanics with nonlinear forces. The contrast between parabolic and realistic trajectories reinforces the importance of modelling assumptions in physics (Cohen et al., 2013).

This study however has some limitations. The study employs constant coefficient  $C_d$  and  $C_l$  whereas in reality, these values vary with velocity, spin and surface roughness. Also, turbulence and seam orientation effects (e.g., “knuckleball” trajectories) are not modelled. Future studies could incorporate empirical measurements or computational fluid dynamics (CFD) simulations for greater accuracy.

## 4 Conclusion

This study investigated the projectile dynamics of a soccer ball under three progressive models:

- a. Ideal projectile motion,
- b. motion with quadratic aerodynamic drag and
- c. motion with both drag and the Magnus effect.

The analysis demonstrates that idealized trajectories significantly overestimates both range and height, predicting unrealistic flight paths compared to actual soccer play. Aerodynamic drag is the dominant resistive force, reducing the effective range by nearly half and lowering the optimal launch angle from the textbook value of  $40^\circ$ -  $45^\circ$  to around  $20^\circ$  -  $25^\circ$  consistent with observed free-kick angles. The Magnus effect introduces substantial lateral deviation,



enabling the “bending” trajectories characteristic of professional free kicks. Moderate spin enhances control and curvature, whereas excessive spin increases drag and diminishes forward range.

The findings highlight the interplay between classical mechanics and fluid dynamics in determining real-world projectile behaviour. From a sports science perspective, this analysis explains the physical basis of soccer free kicks, offering quantitative insight into how players exploit spin and launch conditions to curve the ball around obstacles. From a pedagogical standpoint, the soccer ball provides an engaging, accessible example for illustrating how idealized physics models must be extended with non-linear forces to accurately capture reality.

Further extensions of this study could include experimental validation through high-speed video tracking of free kicks, incorporation of variable drag and lift coefficients dependent on Reynolds number and spin parameter and computational fluid dynamics (CFD) simulations to capture seam and turbulence effects. Such approaches would bridge theoretical modelling with real-world complexity, further enriching both sports physics research and physics education.

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